Some Exotic Phenomena in Quantum World

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God used beautiful mathematics in creating the world.

It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of a mathematical theory of great beauty and power, needing quite a high standard of mathematics for one to understand it. You may wonder: Why is nature constructed along these lines? One can only answer that our present knowledge seems to show that nature is so constructed. We simply have to accept it. **One could perhaps describe the situation by saying that God is a mathematician of a very high order, and He used very advanced mathematics in constructing the universe.**

*Dirac: "Principle of Mathematical Beauty"
we do not really know what the basic equations of physics are, but they have to have a great mathematical beauty. We must insist on that.*

"Beauty is the first test: there is no permanent place in the world for ugly mathematics."
In Exploratory Hall’s tile floor, the circles represent splashes of water droplets, the squiggles are the outline of a double helix, and the Fibonacci spiral shows up cutting through it all. The “Periodic Tables” provide a visual pun. Photo by Alexis Glenn
Those who are not shocked when they first come across quantum theory cannot possibly have understood it.

-Niels Bohr
Quantum Science is one of the most powerful, transformational, and precise tools for exploring nature.

It is also one of the most mysterious and misunderstood aspects of all science.

Developed early in the 20th century to solve a crisis in understanding the nature of the atom, quantum mechanics has laid the foundation for theoretical and practical advances in 21st century physics.

Newtonian or Classical World: Particles & Waves

Quantum World: Wave Particle Duality
Dual Personalities: Waves or particles

Matter Waves

For a baseball of mass 0.5-kg, at a velocity of 50 m/s,

\[ \lambda = \frac{h}{mv} = 2.65 \times 10^{-35} \text{m}, \text{ completely undetectable} \]

For an electron moving with speed 3.00 \times 10^7 \text{ m/s},

\[ \lambda = \frac{h}{m_e v} = 2.43 \times 10^{-11} \text{m}, \text{ about same wave length as X-rays} \]

For Rubidium atoms in thermal equilibrium at temp T,

\[ \lambda_T = \frac{h}{\sqrt{3kmT}} \]

\[ \lambda_{300K} \approx 2 \times 10^{-11} \text{m}, \text{ of the order of atomic size} \]

\[ \lambda_{3\mu K} \approx 2 \times 10^{-7} \text{m}, \text{ of the order of interatomic separation} \]
Bohr Model of Atom
energy quantization

The electron travels in circular orbits around the nucleus. The orbits have quantized sizes and energies. Energy is emitted from the atom when the electron jumps from one orbit to another closer to the nucleus. Shown here is the first Balmer transition, in which an electron jumps from orbit \( n = 3 \) to orbit \( n = 2 \), producing a photon of red light with an energy of 1.89 eV and a wavelength of \( 656 \times 10^{-9} \) m.
Understanding Quantized Orbits

Waves on a String

(a) Waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string bent into a closed circle, we get a rough idea of how electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.
Different kind of Quantization

“quantized Hall conductivity”

Hall conductance have been found to be integer multiples of $e^2/h$ to nearly one part in a billion. This phenomena has allowed for the definition of a new practical standard for electrical resistance, based on the resistance quantum given by the von Klitzing constant $R_K = h/e^2 = 25812.807557$.

Why is it surprising?? When you think of resistance, we know it depends on the material, its width, length...

New state of matter, topological state and integers are topological invariants
Hall Effect
Edwin H. Hall (1879)

Hall resistance $R_H = \frac{V_H}{I}$
Why do we care?

Doing research is hard work and condensed matter systems are particularly opaque. So the question naturally arises: why are we working on this, are we masochists or what? To which the answer is of course, Yes; we are theoretical physicists, remember? But apart from that, there are many reasons to be interested in the quantum Hall effect.

- A very obvious first reason is that a large and growing part of the world's information storage and manipulation depends on the movement of electrons through semiconductors. Everything that could possibly be known about the subject should therefore be known.
- For those who immediately want to know whether something has practical applications: The integer quantum Hall effect is now used as the international standard of resistance. The incredibly accurate quantisation of the Hall resistance to approximately one part in $10^{8}$ makes this possible.
- The constant $e^2/h$ is proportional to the `fine structure constant' in electrodynamics, which basically tells how strongly light interacts with matter. The quantum Hall effect provides an independent way of accurately measuring this constant.
- To a theoretical physicist, the fractional effect is a mouth-watering feast of new theories, nice mathematics, exotic statistics and topology galore. I will try to explain this below.

When Quantization works with very high precision, it forms the basis for precision measurements of magnetic field variations

- Resistance standard since 1990: $h/e^2 = 25812.806$ Ohms (precision $\sim 2 \times 10^{-8}$)
- Fine structure constant: $\alpha = e^2/2\hbar c\varepsilon_0 \approx 1/137.036$ (uncertainty $\sim 0.3$ ppm)

Quantum Hall is not the end, but just the beginning of the story
Understanding Quantized Conductivity

Integer Quantum Hall Effect

How can we understand the remarkable precision of Hall quantization despite the imprecise characterization of the experimental materials, which have different impurities, different geometries?

Answers to above question lie in topology.
The model system that exposed the topological aspect is the system Hofstadter studied in 1976...

Before we discuss topological origin of Quantum Hall effect, we will discuss another interesting phenomena in the Hofstadter model, “FRUSTRATION”
About Douglas R. Hofstadter

- son of Robert Hofstadter who got the nobel prize in 1961 for his research on the structure of atomic nuclei.
- author of the famous book Gödel, Escher, Bach
Hofstadter Model: Energy Spectrum of Electron in a crystal in the magnetic field

Quantum Fractal
Topology and Quantum Hall Effect
“Topological Look at Quantum Hall Effect,”
Aug 2003, Physics Today

Integers in Quantum Hall effect are closely related to Berry phase

ANTICIPATIONS OF THE GEOMETRIC PHASE

The notion that a quantum system’s wavefunction may not return to its original phase after its parameters cycle slowly around a circuit had many precursors—in polarized light, radio waves, molecules, matrices and curved surfaces.

Michael Berry
Beyond Quantum Hall
Quantum Hall system is an example of a Topological Insulator

Topological Insulators are unconventional materials that are insulating in the bulk but conduct along the edges/surfaces. Edge transport is dissipationless.

Magic of Quantum Hall: separate the forward and backward motion spatially so the impurity does not affect it.
Quantum Spin Hall systems is a generalization with spin.
Fractional Quantum Hall Effect, 5/2-state, non-Abelian statistics and Topological Quantum Computing physics today, 2006

New era in condensed matter: New states of matter, just like finding new particle in particle physics
Discovery of the 2D and 3D topological insulator

BiSb Theory: Fu and Kane, PRB 76, 045302 (2007)
Bi$_2$Te$_3$, Sb$_2$Te$_3$, Bi$_2$Se$_3$ Theory: Zhang et al, Nature Physics 5, 438 (2009)
Theory and Experiment Bi$_2$Se$_3$: Xia et al, Nature Physics 5, 398 (2009),
Experiment Bi$_2$Te$_3$: Chen et al Science 325, 178 (2009)
General reading

For a video introduction of topological insulators and superconductors, see

http://www.youtube.com/watch?v=Qg8Yu-Ju3Vw
Some Useful Links

• https://www.youtube.com/watch?v=SpZqmZPtj9I (Steve Girvin U-tube lecture on Quantum Hall effect, 7 minutes)

• https://www.youtube.com/watch?v=Qg8Yu-Ju3Vw (September 10, 2009) Stanford Professor Shoucheng Zhang, discusses a new class of topological states that have been experimentally realized.


• http://www.learner.org/courses/physics/, has a section on quantum physics
CONCLUSIONS (The Tragedy of Hamlet, by Shakespeare):

- There are more thing in heaven and earth, Horatio, than are dreamt of in your philosophy.

Wow!!!
MOSFET-Metal Oxide Semiconductor Field Effect Transistor

The experiments show that between two adjacent Landau levels, the Hall resistance has fixed values and the longitudinal resistance $R_{xx}$ vanishes, which means that the electrons are localized in this region. Localization is a key point to interpret IQHE.

Due to impurity, the density of states will evolve from sharp Landau levels to a broader spectrum of levels\(\text{(Figure 7)}\). There are two kinds of levels, localized and extended, in the new spectrum, and it is expected that the extended states occupy a core near the original Landau level energy while the localized states are more spread out in energy. Only the extended states can carry current at zero temperature. Therefore, if the occupation of the extended states does not change, neither will the current change. An argument due to Laughlin\(1981\) and Halperin\(1982\) shows that extended states indeed exist at the cores of the Landau levels and if these states are full, i.e., the Fermi level is not in the core of extended states, then they carry exactly the right current to give Eq. (22).

The existence of the localized states can explain the appearance of plateaus. As the density is increased (or the magnetic field is decreased) the localized states gradually fill up without any change in occupation of the extended states, thus without any change in the Hall resistance. For these densities the Hall resistance is on a step in the Figure and the longitudinal resistance vanishes (at zero temperature). It is only as the Fermi level passes through the core of extended states that the longitudinal resistance becomes appreciable and the Hall resistance makes its transition from one plateau step to the next.

Finally, at finite temperature there is a small longitudinal resistance due to hopping processes between localized states at the Fermi level.