A Greenhouse Gas Illustration

This illustration is taken from Chapter 3 of David Archer, Global Warming. Understanding the Forecast (2012), but the values for top-of-the-atmosphere insolation are the ones used in class: 342 watts/m_2 and 30% albedo. The calculations make use of the Stephan-Boltzmann equation that is footnoted at the bottom of Class 1 Slide 17: radiation = σT^4 where radiation is in watts/m^2, temperature is in degrees Kelvin (centigrade temperature + 273) and $\sigma = 5.67 \times 10^{-8}$ watts.

With albedo of 30%, the energy warming the surface is 70% of 342 watts/m_2 or 239 watts/m^2, and at equilibrium this must equal the outgoing thermal radiation. The task is to find the temperature that produces that much radiation. For this, rearranging the terms in the equation gives: $(342/(5.67 \times 10^{-8})^{1/4} = 255^o$ K., or -18^o C.

Now, rather than adding a greenhouse gas, suppose there is a sheet of glass suspended in the atmosphere, so that all incoming and outgoing radiation must past through it. The glass is transparent to the incoming high-energy photons from the sun but it absorbs all of the outgoing lower-energy photons that make up Earth's thermal radiation. Like a greenhouse gas, the glass then immediately re-emits a photon of the same energy but in a random direction. Hence, half of the re-emitted photons go upward and out into space and half go downward to strike Earth's surface.

At equilibrium, the energy of the photons that go into space must equal the energy in the incoming solar radiation that is not reflected, or 239 watts/m^2. We have already calculated the temperature corresponding to that radiation. It is 255^o K. With this "glass-sheet model", this is the outgoing thermal radiation that someone on Mars would observe and the temperature they would calculate.

Since the glass re-emits photons in a random direction, (1) the other half would go downward to the Earth's surface and (2) their energy would be the same as those escaping into space. The energy warming the Earth therefore would be the unreflected incoming solar radiation (239 watts/m^2) plus downward re-emissions from the glass (also 239 watts/m^2), for a total of 478 watts/m^2, and at equilibrium this must also be the amount of thermal radiation. Again using a rearranged Stephan-Boltzmann equation gives: $(478/(5.67 \times 10^{-8}) = 303^o$ K. This is about 10^o too high but much closer than the 255^o calculated without the greenhouse effect

Just to tie things up, the 478 watts/m^2 leaving Earth's surface strikes the glass. Nothing is lost, but only half of it continues upward. This is 239 watts/m^2, which balances the books with the unreflected top-of-the-atmosphere solar radiation.

In reality, the greenhouse gases in Earth's atmosphere do not absorb all of the outgoing thermal radiation.[1] However, the glass-sheet model illustrates how greenhouse gases increase the average temperature of Earth's surface and why that temperature is higher than the one that would be measured from space.

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[1] For a time it was argued that the calculations of Arrhenius were incorrect because the atmosphere was already saturated with greenhouse gases. It turned out that it was the calculations on which this argument was based that were incorrect.