The History of Mathematics From the Egyptians to Archimedes

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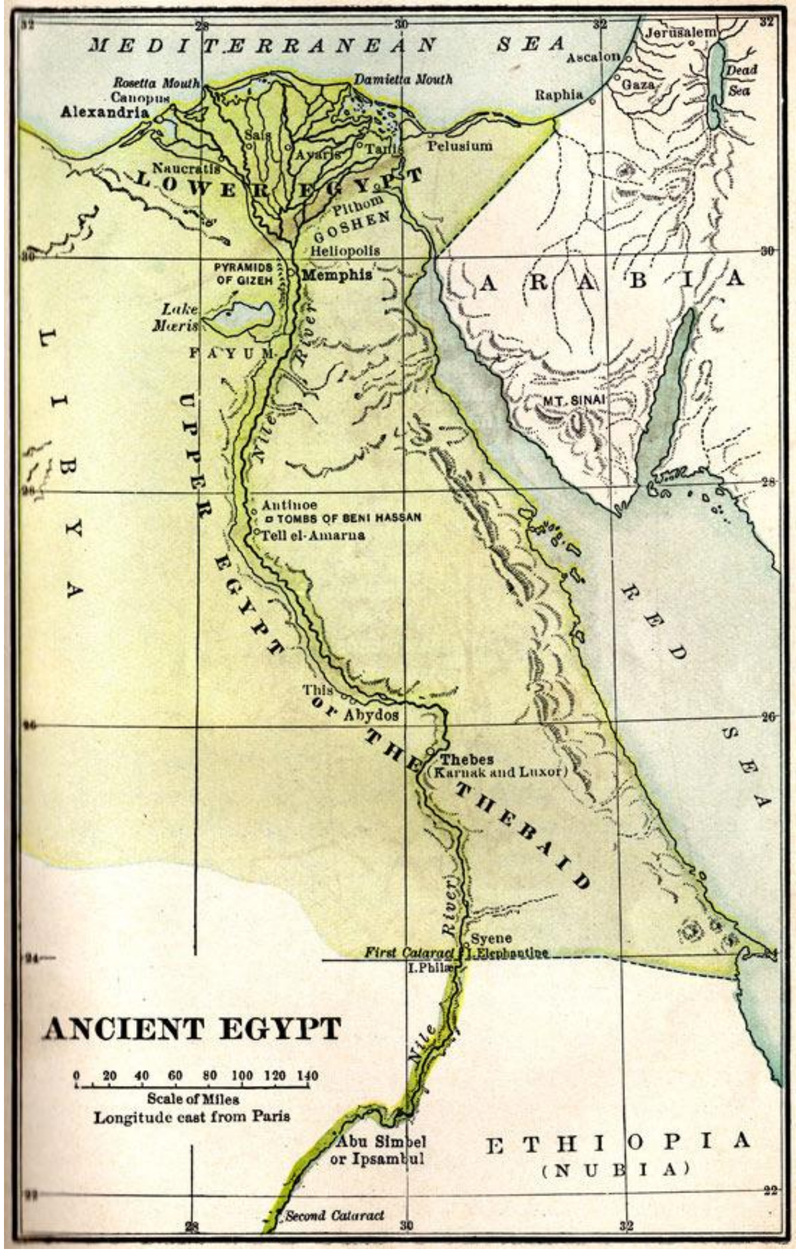
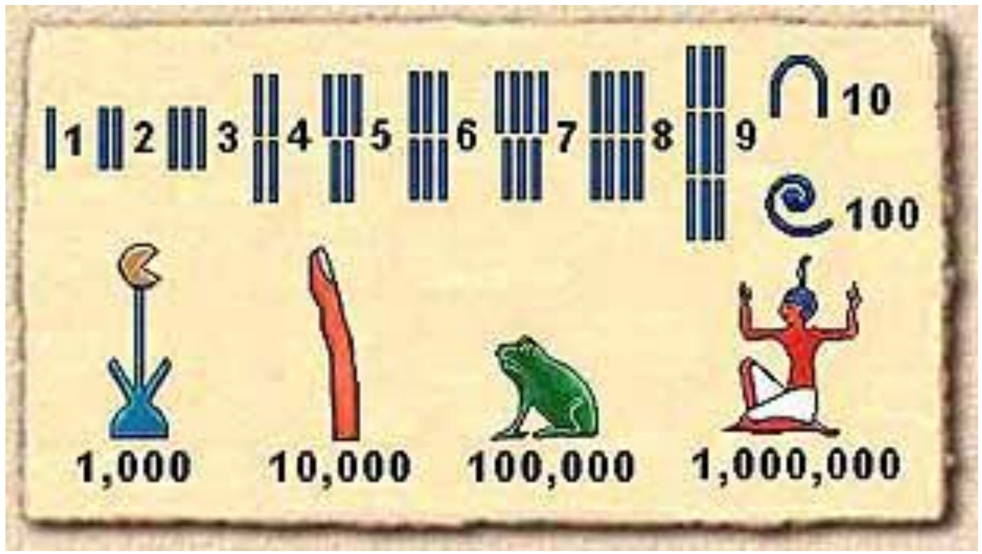
OLLI Winter 2011

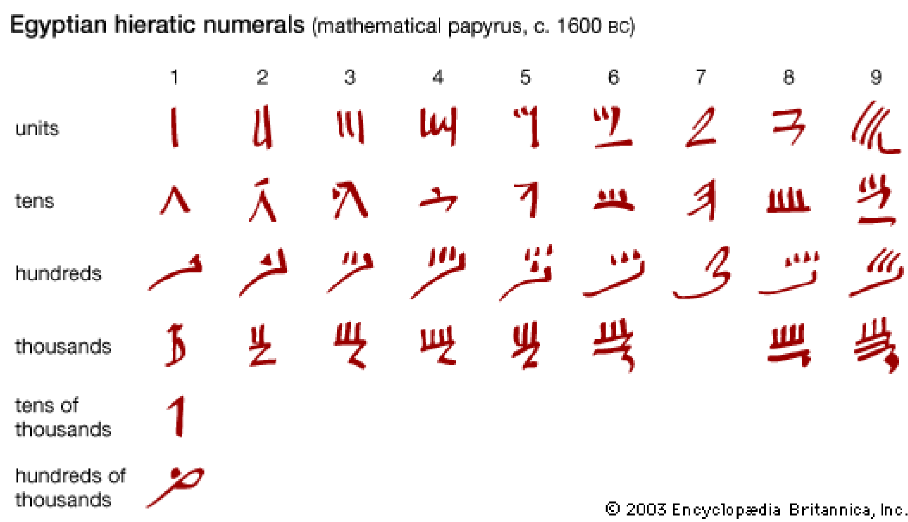
Egyptian and Babylonian

- 3000 BC to 260 AD
- Essentially empirical
- Early number systems
- Simple arithmetic, practical geometry
- Egyptian papyri & Babylonian cuneiform tablets
  - Mathematical tables
  - Collections of mathematical problems

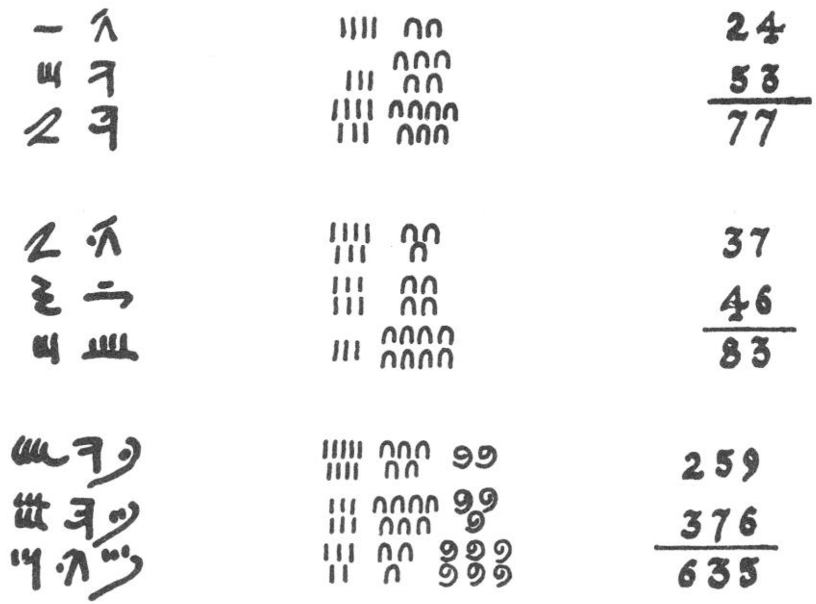

MEDITERRANEAN SEA
Jerusalem
Dead Sea
Ascalon
Gaza
Raphia
Rosetta Mouth
Canopus
Alexandria
Damietta Mouth
Sais
Avaris
Tanis
Pelusium
Naucratis
LOWER EGYPT
Pithom
GOSHEN
Heliopolis
PYRAMIDS OF GIZEH
Memphis
ARABIA
Lake Mœris
FAYUM
LIBYA
UPPER EGYPT
Nile River
MT. SINAI
Antinoe
TOMBS OF BENI HASSAN
Tell el-Amarna
RED SEA
This or Abydos
Thebes
(Karnak and Luxor)
THE THEBAID
Syene
First Cataract
I. Elephantine
I. Philæ
ANCIENT EGYPT
0 20 40 60 80 100 120 140
Scale of Miles
Longitude east from Paris
Abu Simbel or Ipsambul
ETHIOPIA
(NUBIA)
Second Cataract

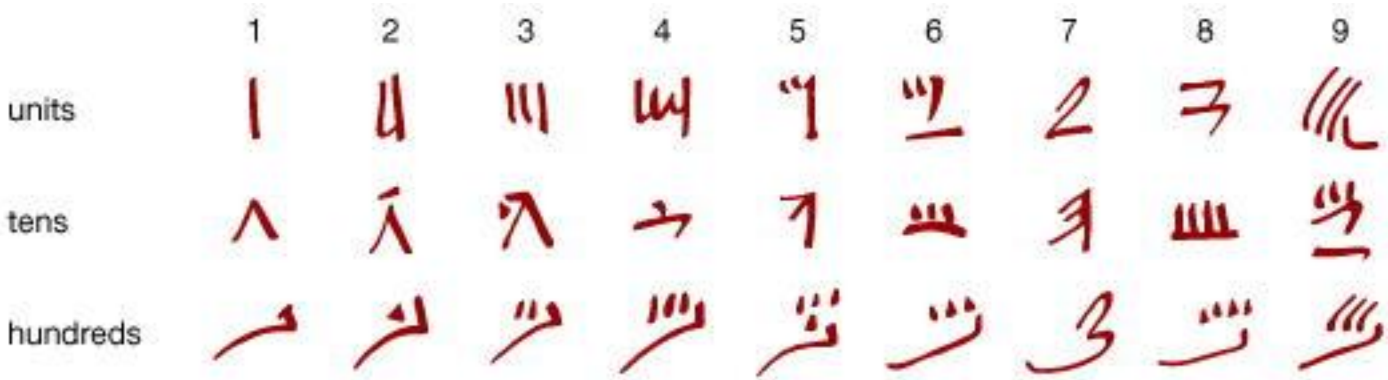
Hieroglyphic & Hieratic Numerals





Hieratic & Hieroglyphic





Egyptian Arithmetic

- The Egyptians could add, subtract, multiply and divide
- Multiplication of whole numbers used the method of doubling

Example: 57 x 117

1+	117
2	234
4	468
8+	936
16+	1872
32+	3744
57	6669

Primary Egyptian Sources

- Rhind Mathematical Papyrus (RMP)
  - About 1650 BC from writings made 200 years earlier (18 ft x 13 in)
  - The Recto Table and 84 (87) mathematical problems
- Moscow Mathematical Papyrus –
  - 1850 BC, (18 ft x 1.5 to 3 in)
  - 25 problems

The Recto Table

- The division of 2 by the odd numbers 3 to 101 with the answers expressed as the sum of unit fractions
- The entrance into the knowledge of all existing things and all obscure secrets. This book was copied in the year 33, in the 4th month of the inundation season, under the majesty of the king of Upper and Lower Egypt, A-user-Re (Aweserre Apopi), endowed with life, in likeness to writings of old made in the time of Upper and Lower Egypt, Ne-ma-et-Re (Nemare Ammenemes III). It is the scribe Ah-mose who copies this writing.

Egyptian Fractions & Algebra

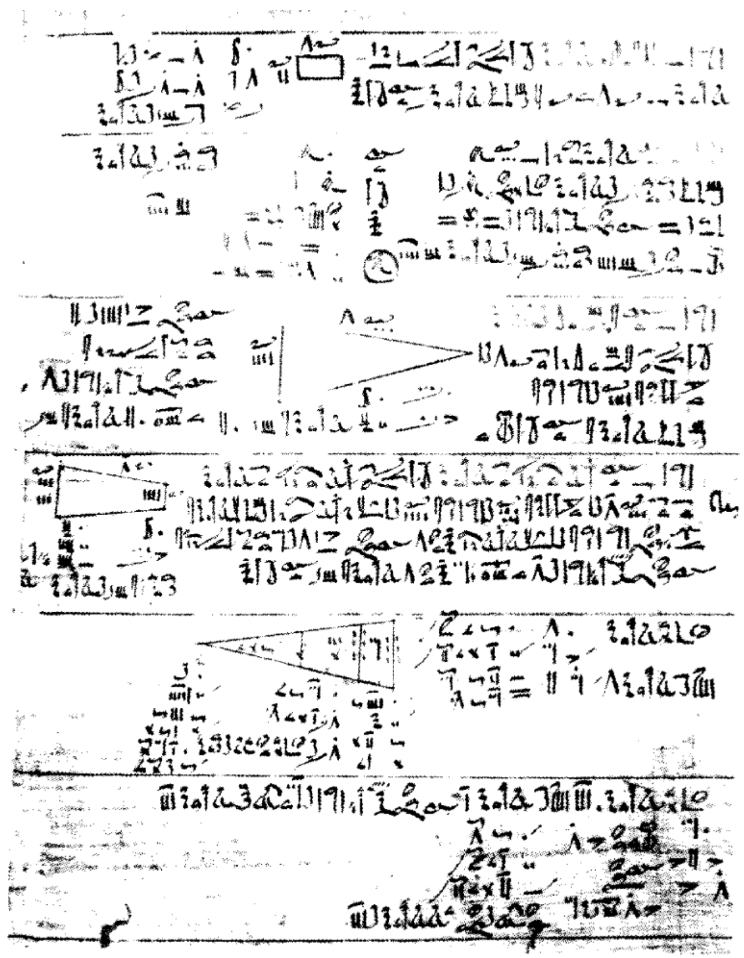
- Except for 2/3 and possibly 3/4 the Egyptian arithmetic notation only permitted fractions with unity in the numerator
- 2/n equivalents in the RMP Recto table

  5: 3 15   7: 4 28   9: 6 18   11: 6 66   13: 8 52 104

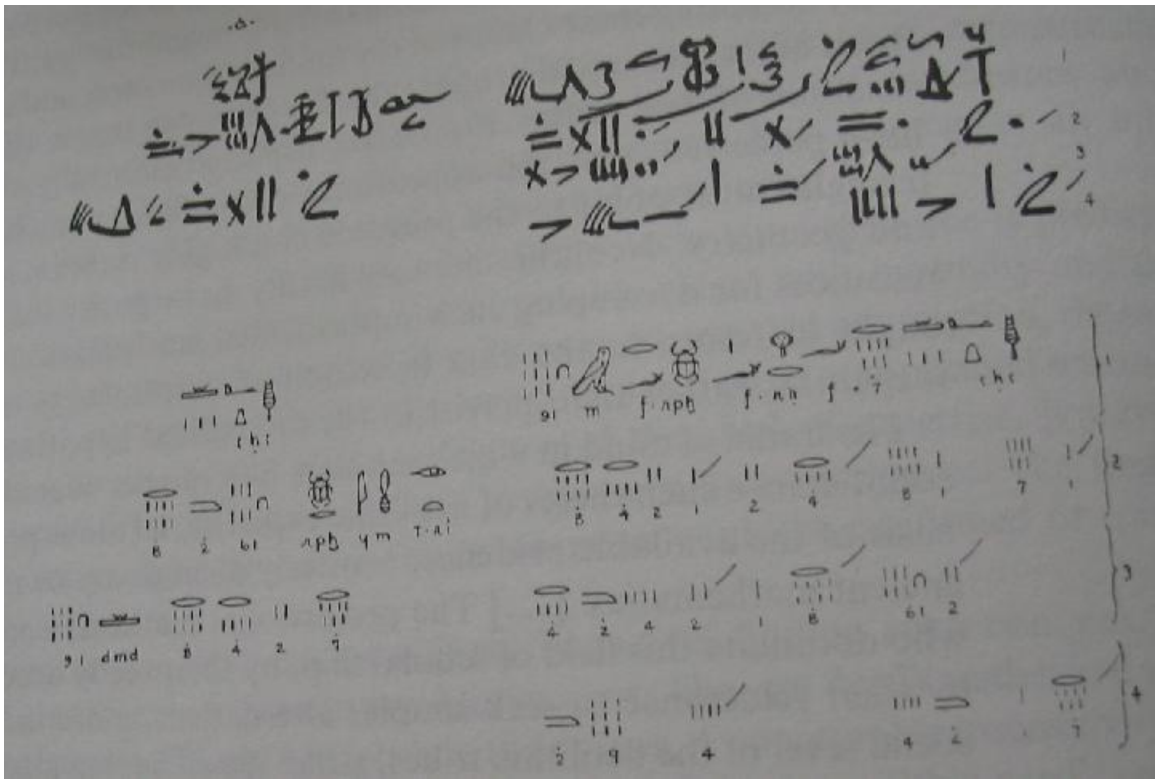
  15: 10 30   17: 12 51 68   19: 12 76 114   21: 14 42

  23: 12 276   25: 15 75 …   97: 56 679 776 …
- Could multiply and divide mixed numbers
- Simple Algebra
  - A quantity and its seventh, added together give 19. What is the quantity. (Method of single false position)
  - The sum of the areas of two squares is 100. Three times the side of one is four times the side of the other. Find the sides of the squares.
  - Arithmetic series and geometric series

Rhind Papyrus



Problem 24 of RMP



Hieratic

Hieroglyphic

Egyptian Geometry

- Area and perimeter
  - Rectangle, triangle, trapezoid, arbitrary quadrilateral, circle
  - Rule for arbitrary quadrilateral was incorrect
  - Area of circle given by the area of a square with side 8D/9 where D is the circle diameter. Implies $\pi \approx 3.16$.
  - Egyptians knew that the ratio area/perimeter for a circle and a circumscribed square are equal. (some debate)
  - May have known the surface area of a sphere
- Volume
  - Pyramid, truncated pyramid

Egyptian Mathematics

- The Egyptian papyri show practical techniques for solving everyday problems
- The rules in the papyri are seldom motivated and the papyri may in fact only be manuals for students
- However, they demonstrated a solid understanding of the operations of addition, subtraction, multiplication and division and enough geometry for their needs
- There was little change in their approach for at least 1000 years

The Babylonian Empire
Caspian Sea
Nineveh
ASSYRIA
Carchemish
Sidon
Tyre
Damascus
CYPRUS
SYRIA
Samaria
JUDAH
Jerusalem
MOAB
EDOM
Babylon
BABYLONIA
Susa
Ur
ELAM
PERSIA
EGYPT
Memphis
Persian Gulf
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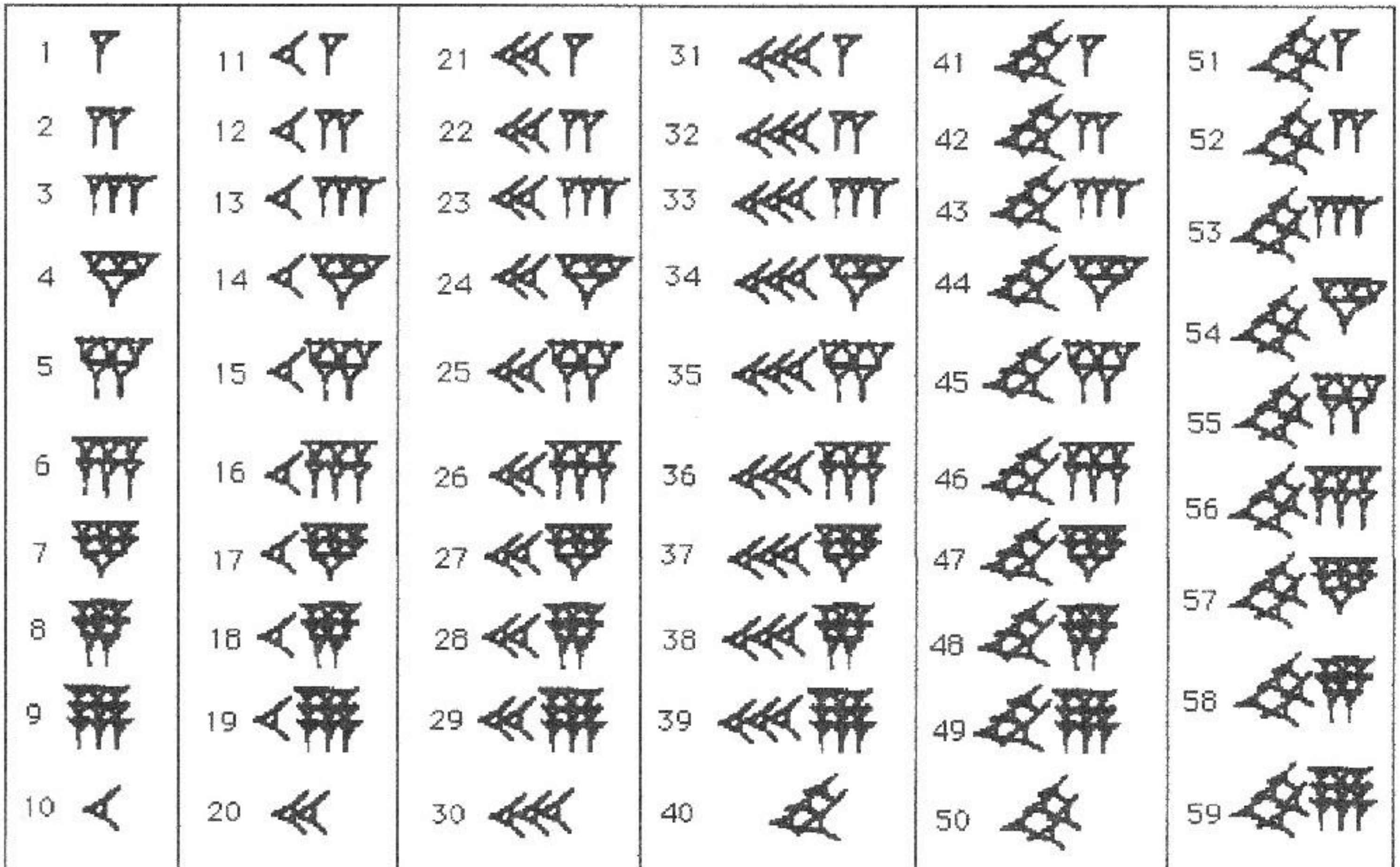
Mesopotamian Mathematics

- Babylonian civilization (2000 BC – 600 BC) replaced Sumerian and Akkadian civilizations
- Base 60 positional number system
- Numbers system built on symbol for 1 and 10
- No zero and no character to indicate a position was vacant until 300 BC
- More general treatment of fractions than Egyptians
- Great advance was that their number system was extended to fractions and positional notation.

Symbols for numbers 1 – 59



Number System Examples

- Use ‘I’ for the Babylonian one and ‘<‘ for the Babylonian 10
- <<II = 22, <<<IIII = 34
- << << = 20*60 + 20 = 1220
- << << = 20*60^2 + 20*60 = 73200
- II II <I = 2*60 + 2 + 11/60
- I II could be 62 or 3602

In approximately 300 BC the Babylonians added a symbol like “ to show if a position was empty. It was only used for intermediate position and not end positions (no true zero)

- I “ II means 1*60^2 + 0*60 + 2 or 60 + 0 + 2/60

Multiplication

- Babylonians could multiply in their base 60 system the same way we multiply in our base 10 system

Base 10	Base 60	
139	2,19	
x72	1,12	12x19 = 3,48 (from table)
278	27,48	
973	2,19	
10008	2,46,48	$= 2x60^2 + 46x60 + 48 = 10008$

Other Features

- Developed an accurate process for finding square roots

  divide & average $\sqrt{2} = 1.41421296$

  correct $\sqrt{2} = 1.41421356$

- Constructed tables of squares to assist in multiplication

  $$xy = ((x + y)^2 - (x - y)^2)/4$$

- Able to solve quadratic equation and linear equations in two variables
- Solved cubic equations using tables and interpolation
- Arithmetic and geometric progressions

Quadratic Equation Prescription

Problem Statement: Find two numbers with sum 20 and product 91

1) Find half of the sum : 10
2) Subtract the product from the square of result of step 1:

   100 – 91 = 9

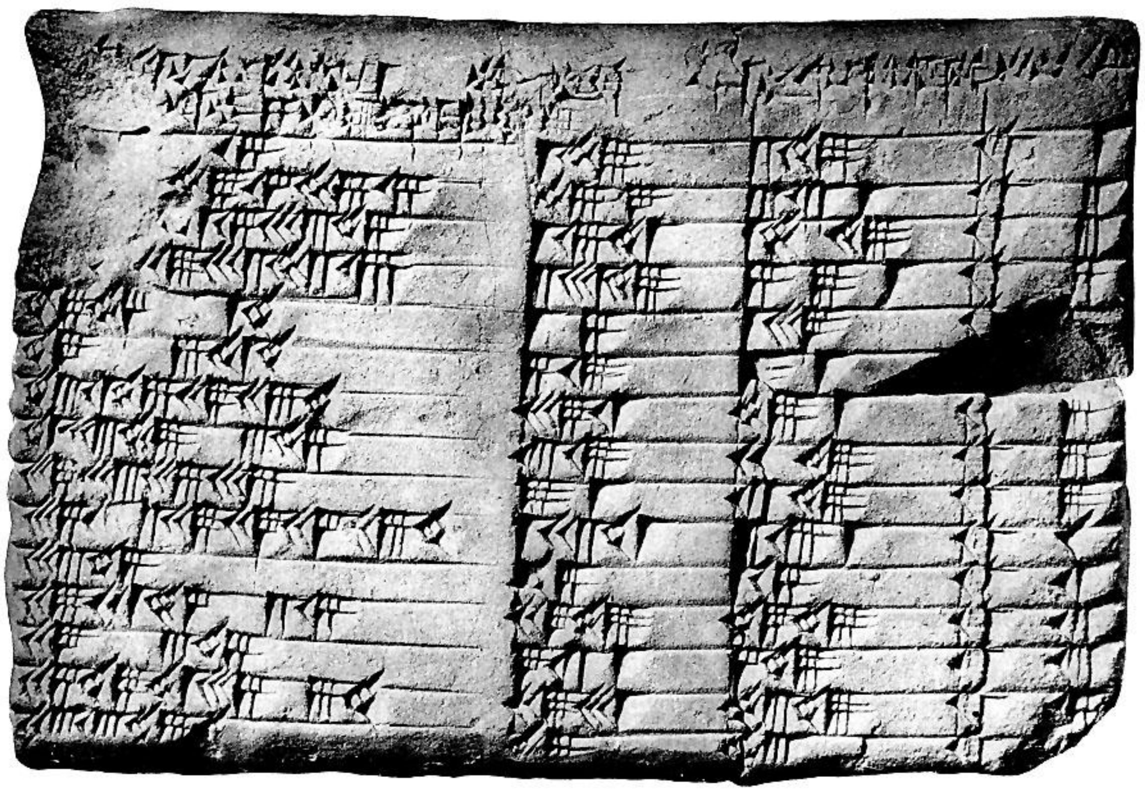
3) Take the square root of the result of step 2: 3
4) Add and subtract the result of step 3 to the result of step 1:

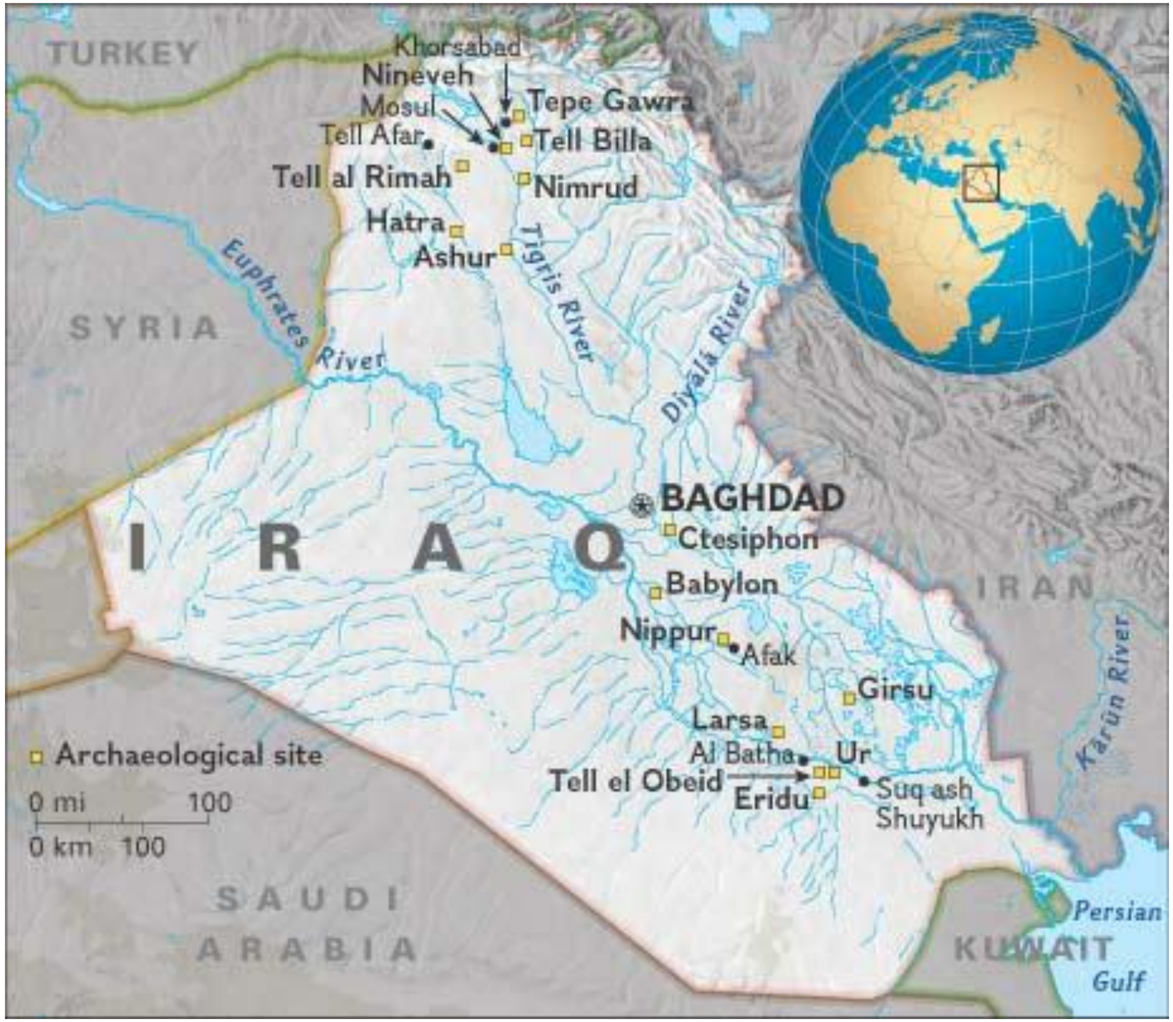
   10 + 3 = 13 and 10 – 3 = 7

5) The numbers are 13 and 7

Babylonian Geometry

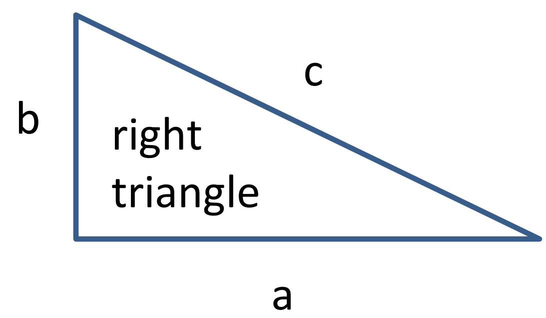
Plimpton 322 Tablet




TURKEY
SYRIA
IRAQ
IRAN
SAUDI ARABIA
KUWAIT
Persian Gulf
Khorsabad
Nineveh
Mosul
Tepe Gawra
Tell Afar
Tell Billa
Tell al Rimah
Nimrud
Hatra
Ashur
Euphrates River
Tigris River
Diyala River
BAGHDAD
Ctesiphon
Babylon
Nippur
Afak
Girsu
Larsa
Al Batha
Ur
Tell el Obeid
Eridu
Suq ash Shuyukh
Karun River
Archaeological site
0 mi 100
0 km 100

Pythagorean Triplets

A Pythagorean triple is a set of three integers such that $a^2 + b^2 = c^2$



Example : $3^2 + 4^2 = 5^2$

Plimpton 322 contains 15 sets of three numbers. The 2nd and 3rd numbers are part of triplets

Example (2nd line): $3456^2 + 3367^2 = 4825^2$

4825 = 1:20:25

The 1st number is a fraction related to two of the sides of the triangle

Generally believed that the Babylonians knew the Pythagorean Theorem

Geometry not a Babylonian Strength

For practical problems they calculated the area of a circle by $A = C^2/12$ where C is the circumference. This implies a value $\pi = 3$. Tablets found in Susa included the ratio of the perimeter of a hexagon to the circumference of the circumscribed circle and implied a value of $\pi = 3.125$. Babylonian geometry was a collection of rules for the areas of plane figures and the volumes of simple solids.

Greek/Hellenistic Timeline

- **Classical Period (500-336 BC)** - Classical period of ancient Greek history, is fixed between about 500 B. C., when the Greeks began to come into conflict with the kingdom of Persia to the east, and the death of the Macedonian king and conqueror Alexander the Great in 323 B.C. In this period Athens reached its greatest political and cultural heights: the full development of the democratic system of government under the Athenian statesman Pericles; the building of the Parthenon on the Acropolis; the creation of the tragedies of Sophocles, Aeschylus and Euripides; and the founding of the philosophical schools of Socrates and Plato.
- **Hellenistic Period (336-146 BC)** - period between the conquest of the Persian Empire by Alexander the Great and the establishment of Roman supremacy, in which Greek culture and learning were pre-eminent in the Mediterranean and Asia Minor. It is called Hellenistic (Greek, Hellas, "Greece") to distinguish it from the Hellenic culture of classical Greece.


The Beginnings of Historic Greece
700 - 600 B.C.
Area occupied by tribes
Area occupied by cities
Political leagues
Great states
Delian Amphictyony
Delphic Amphictyony
Scale 1: 7 500 000
Miles
THRACIAN SEA
PROPONTIS
THESSALY
AETOLIA
ACHAIA
ELIS
ARCADIA
ARGOLIS
ATTICA
Athens
MESSENIA
LACEDAEMON
Cythera
Tenedos
Ida Mt.
Lesbos
Chios
Samos
Miletus
Mt. Tmolus
Halicarnassus
Cnidus
Rhodes
Delos
Ambracian Gulf
Long. East 24 of Greenwich


The Hellenistic World
cities and empires in 240 BCE
MACEDONIA
AETOLIA
ACHAEA
ATTALID
BITHYNIA
GALATIA
ARMENIA
ATROPATENE
CAPPADOCIA
CILICIA
CRETE
RHODES
EGYPT
Nicomedia
Pergamum
Ephesus
Antioch
Dura-Europos
Seleuceia
Babylon
Damascus
Tyre
Alexandria
Gaza
Petra
Major Cities
Antigonid Empire
Independent States
Seleucid Empire
Ptolematic Empire

Early Greek Number System

- Acrophonic (Attic, Herodianic)

Γ	Δ	Η	Χ	Μ
Pente	Deka	Hekaton	Khilioi	Murioi
Πεντε	Δεκα	Ηεκατον	Χιλιοι	Μυριοι
5	10	100	1000	10000

~ 600 BC to ~ 300 BC

Similar to Roman Numerals

Ι	ΙΙ	ΙΙΙ	ΙΙΙΙ	Γ	ΓΙ	ΓΙΙ	ΓΙΙΙ	ΓΙΙΙΙ	Δ
1	2	3	4	5	6	7	8	9	10
1 - 10 in Greek acrophonic numbers									

𐅆𐅅Η𐅄ΔΔΓ⊢⊢⊢
5678 drachma

Δ	𐅄	Η	𐅅	Χ	𐅆	Μ	𐅇
10	50	100	500	1000	5000	10000	50000
Higher numbers and combining acrophonic numerals							

Denotes drachma

Later Greek Number System

- Ionic (Alphabetical)

Α	Β	Γ	Δ	Ε	Ϛ	Ζ	Η	Θ
α	β	γ	δ	ε	ϛ	ζ	η	θ
1	2	3	4	5	6	7	8	9

Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π	Ϙ
ι	κ	λ	μ	ν	ξ	ο	π	ϙ
10	20	30	40	50	60	70	80	90

ια	ιβ	ιγ	ιδ	ιε	ιϛ	ιζ	ιη	ιθ
11	12	13	14	15	16	17	18	19

Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω	Ϡ
ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

͵Α	͵Β	͵Γ	͵Δ	͵Ε	͵Ϛ	͵Ζ	͵Η	͵Θ
1000	2000	3000	4000	5000	6000	7000	8000	9000

$$ {}^{,}\epsilon\chi o\eta = 5678 $$

Fractions

- Acrophonic
  - Essentially the same as the Egyptians
- Ionic
  - Used apostrophe to signify fraction

    γ' = 1/3 $\lambda\beta'$ = 1/32

    $\bar{\iota}\ \ o\alpha'$ = 10/71 (Archimedes)
- Addition, subtraction, multiplication and division done the same way we do it

Greek Mathematics

Roughly 600 BC to 300 AD

We will cover 600 BC to 200 BC

How do we know about Greek mathematics?

- Initially knowledge was passed from teacher to student orally
- Probably around 450 BC chalk boards and wax tablets were introduced for non permanent work.
- Papyrus rolls were used for permanent records but new copies were required frequently.
- In about 300 BC Euclid’s Elements was completed and it was so comprehensive and of such quality that all older mathematical texts became obsolete.
- In 2nd century AD books of papyrus appeared and became the main form in the 4th century. Also vellum (animal skin) was introduced.

How do we know about Greek mathematics?

- If the person copying the "Elements" was a mathematician, material not in the original text may have been added
- The oldest surviving complete copy of the Elements is from 888 AD probably based on a version with commentary and additions produced by Theon of Alexandria in the 4th century AD
- In addition to the 888 AD document there are numerous fragments some dating to as early as 225 BC
- Some surviving texts exist that were written after 888 AD that were based on versions of the elements earlier than 888 AD
- First print edition in Venice in 1482
- For the complete story see the article "How do we know about Greek mathematics" on the Mac Tutor History of Mathematics

Greek Mathematics – Thales of Miletus

- Thales of Miletus 624 BC – 547 BC (Turkey)
  - None (if there were any) of his writings survive
  - Name appears in the writings of others years later
  - The first of the seven sages of antiquity
  - A pupil of the Egyptians
  - Credited with five theorems of elementary geometry
    - A circle is bisected by any diameter
    - The base angles of an isosceles triangle are equal
    - The angles between two intersecting straight lines are equal
    - Two triangles are congruent if they have two angles and a corresponding side equal
    - An angle in a semicircle is a right angle

Greek Mathematics – Pythagoras of Samos

- Pythagoras (569 BC – 475 BC)
  - Another very obscure figure. Biographies written in antiquity all lost
  - Founded a school that was both communal and secret
  - Difficult to separate history and legend
  - The Pythagoreans examined the basic principles of mathematics and gave it an intellectual structure.
  - Major mathematical influence on Pythagoras likely to have been Anaximander, a pupil of Thales , and the Egyptians
  - The Pythagoreans taught that the purpose of life was to purify the soul and body. To reach purification one had to discover the “harmonies” of the cosmos – and scientific (mathematical) inquiry was the vehicle with which to find them.

All is Number

- One of the scientific achievements of Pythagoras was the discovery of the mathematical order in the musical scale and the harmonies so produced. It is believed that he experimented with string instruments and discovered that two tones sound well together when the ratios of their frequencies can be expressed by the use of small numbers and the smaller the numbers the better is the consonance.

Interval	Frequency ratio	Largest number occurring in ratio
Unison	1 : 1	1
Octave	2 : 1	2
Fifth	3 : 2	3
Fourth	4 : 3	4
Major Third	5 : 4	5
Major Sixth	5 : 3	5
Minor Third	6 : 5	6
Minor Sixth	8 : 5	8
Second	9 : 8	9

Table from “Science & Music” by Sir James Jeans

Pythagorean Scale

Pythagorean frequency ratio	Pythagorean interval	Equal temperament frequency ratio (p. 177)
C $= 1{\cdot}0000$		1·000
	Tone	
D $\frac{9}{8} = 1{\cdot}1250$		1·1225
	Tone	
E $\frac{81}{64} = 1{\cdot}2656$		1·2599
	Hemitone	
F $\frac{4}{3} = 1{\cdot}3333$		1·3348
	Tone	
G $\frac{3}{2} = 1{\cdot}5000$		1·4983
	Tone	
A $\frac{27}{16} = 1{\cdot}6875$		1·6818
	Tone	
B $\frac{243}{128} = 1{\cdot}8984$		1·8877
	Hemitone	
C $= 2{\cdot}0000$		2·0000

Table from “Science & Music” by Sir James Jeans

All is Number

- Boethius (~ 500 CE) tells us that Pythagoras investigated the relation between the length of a vibrating string and the musical tone it produced. If a string was shortened to ¾ of its original length, then what is called the fourth of the original tone was heard; if shortened to 2/3, the fifth was heard; and if shortened to 1/2 , the octave. The string lengths were proportional to 12, 9, 8, and 6.
- The Pythagorean school was able to find many interesting relations between these number including the fact that a cube has 6 faces, 8 vertices, and 12 edges. They were able to convince themselves that since these combinations of string lengths produced sounds that harmonize, the numbers themselves essentially caused it. The Pythagoreans extrapolated to the natural numbers being fundamental to natural science.
- *This was bad science but really good for number theory*

Pythagorean "Arithmetica"

- Pythagorean triplets – natural numbers that satisfy

  $$a^2 + b^2 = c^2$$

- Pythagoreans knew that

  $$(m^2 - 1)^2 + (2m)^2 = (m^2 + 1)^2$$

  produced triplets. For example for m = 2, the triplet is 3, 4, 5. However the formula does not produce all triplets.

  Euclid solves the general problem is his Elements.

More “Arithmetica”

- Perfect numbers – the number is the sum of its divisors.

  6 = 1 + 2 + 3

  28 = 1 + 2 + 4 + 7 + 14

  496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248

  8128

  33550336

  The search for perfect numbers continues to today. No odd perfect numbers have been found.

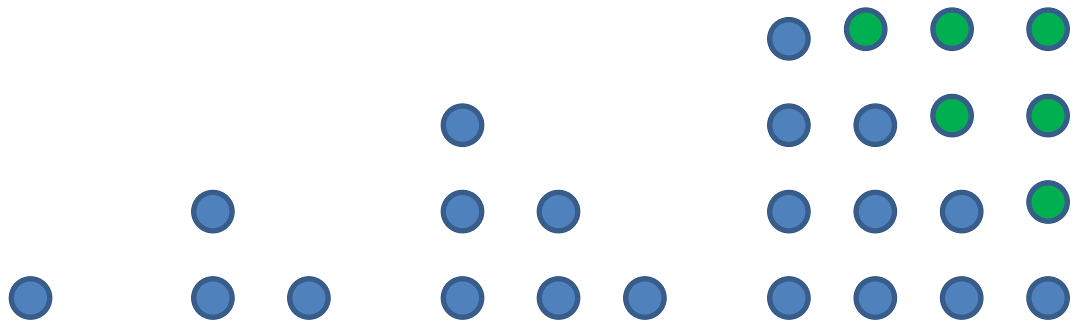
- Friendly numbers – pairs of numbers such that each is the sum of the divisors of the other –

  The first friendly pair is 220 & 284

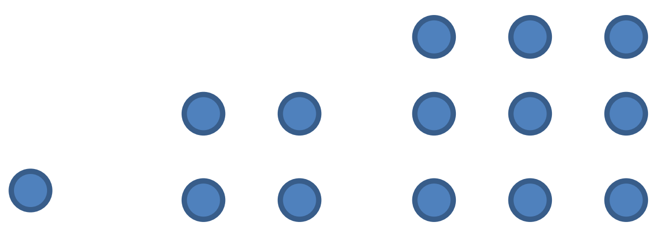
  The secondly pair is 17,296 and 18,416 and was independently discovered by al-Banna (Arab 1256 – 1321) and by Fermat in 1636

  In the 1970s the phrase “friendly numbers” started to be used differently

Figurate Numbers



Triangular Numbers   1, 3, 6, 10, 15 …
Square Numbers   1, 4, 9, 16, 25 …
The sum of two consecutive triangular numbers is a square number



The sum of any number of consecutive odd integers, starting with one, is a perfect square

More “Arithmetica”

- The Pythagoreans studied prime numbers, progressions, and ratios and proportions. They understood that certain sums could be easily calculated.

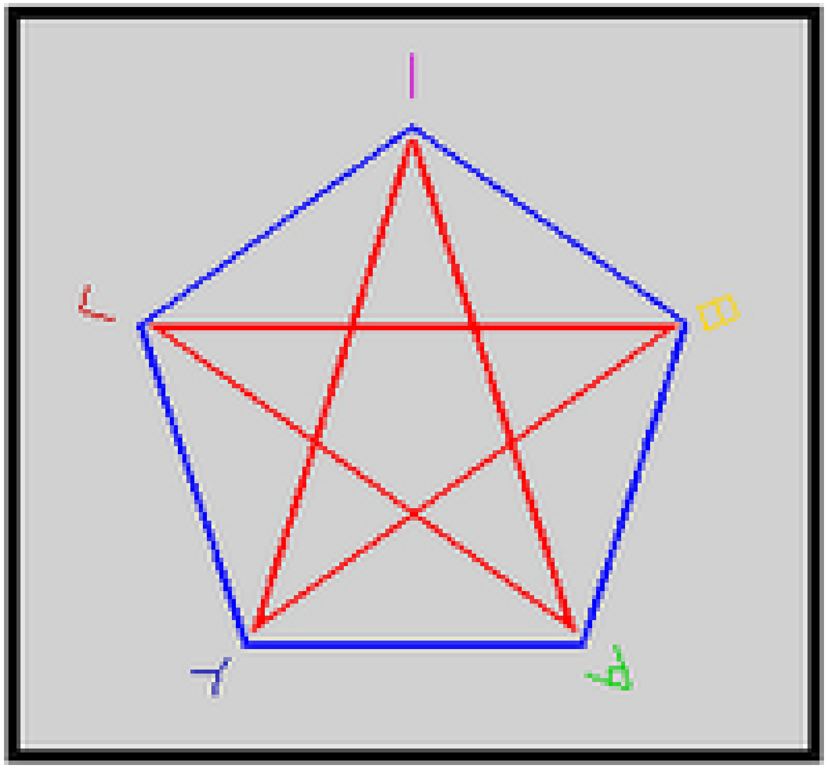
  $1 + 2 + \ldots + n = (n/2)(n + 1)$

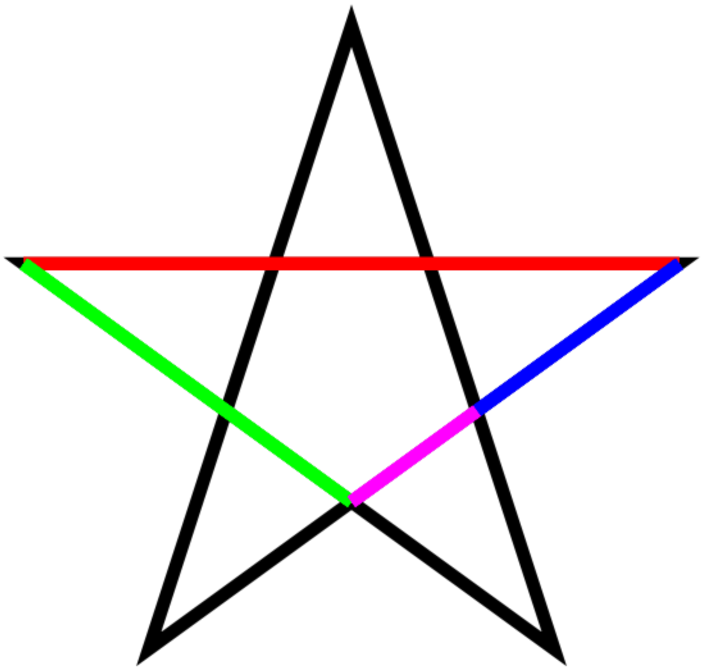
- Numbers to Pythagoreans were whole numbers only. The ratio of two whole numbers was not a fraction and therefore another kind of number. Actual fractions were employed in commerce.
- The most developed part of the *arithmetica* was the theory of even and odd
  - The sum of two even numbers is even
  - The product of two odd numbers is odd
  - When an odd number divides an even number, it also divides its half

Pythagorean School Geometry

- In the 5th century BC geometry, the theorem/proof logic took root and a system of theorems was developed where theorems were proved based on theorems previously proved.
- Studied triangles, parallel lines, polygons, circles, spheres and regular polyhedra
- They knew that the sum of the angles of a triangle equals two right angles
- Worked on a class of problems known as “application of area”
  - Example – Construct a polygon equal in area to a given one and similar in shape to another given one.

Pythagoreans' Symbol

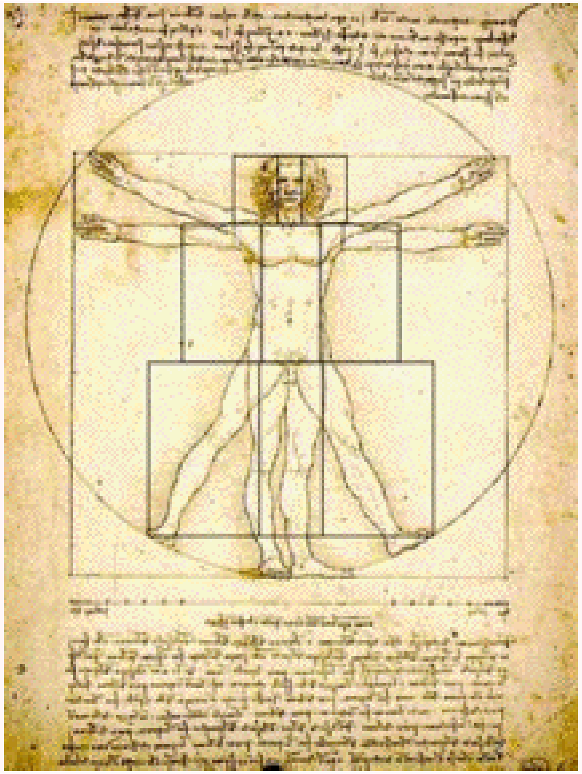


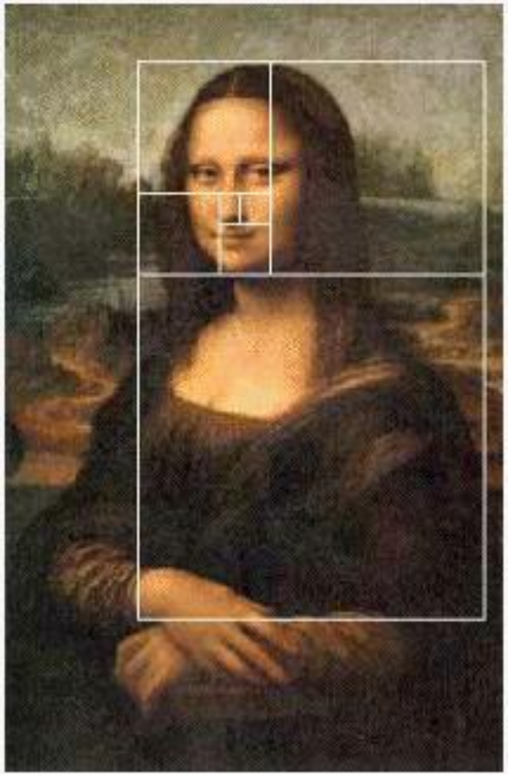


$$\frac{\text{red}}{\text{green}} = \frac{\text{green}}{\text{blue}} = \frac{\text{blue}}{\text{magenta}} = \varphi.$$

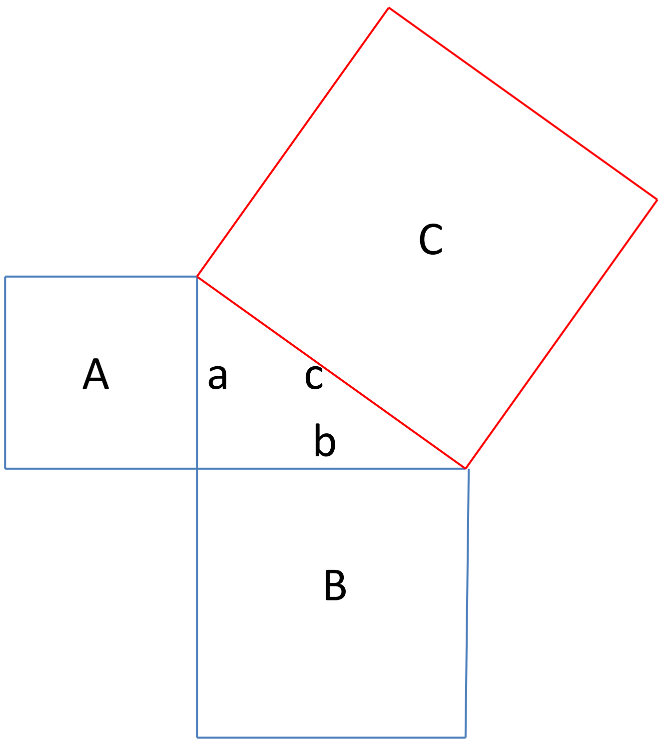
Golden Ratio

Golden ratio and Golden Rectangle



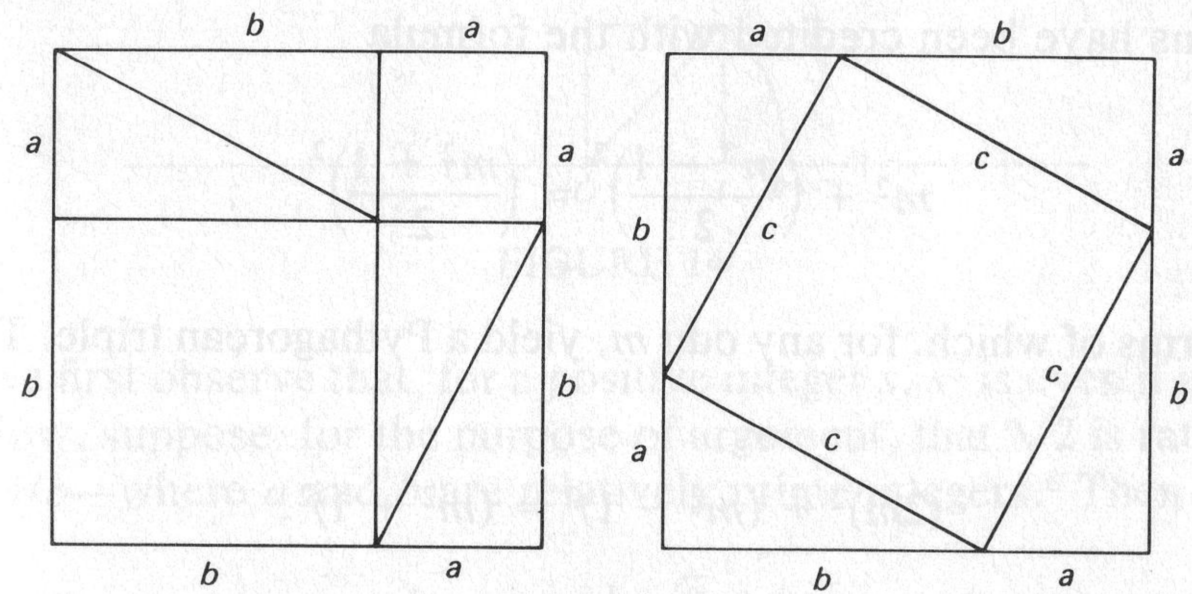


Pythagorean Theorem



For a right triangle with arms **a** and **b** and hypotenuse **c**, then the area of the square **A** constructed on **a** plus the area of the square **B** constructed on **b** equals the area of the square **C** constructed on **c.**

Proof of Pythagorean Theorem



The areas of the two large squares are equal. The area of the square on the left is 4 triangles + square A + square B. The area of the square on the right is 4 triangles + square C. Since all eight triangles are identical,

The area of A + the area of B = the area of C

Rational Numbers

- The concept of integers comes from counting objects
- Life requires us to measure quantities such as length, speed, weight, time , …
- To satisfy these measurements fractions (ratios) are required. No matter how accurately it is necessary to measure something, that measurement can always be expressed as a fraction of the unit of measure.
- Early mathematicians thought of numbers as points on a line and believed that all points could be expressed as fractions.

Irrational Numbers

- From the Pythagorean theorem it follows that for an isosceles right isosceles triangle with arm equal to 1 the hypotenuse equals $\sqrt{2}$. The Pythagoreans tried to find two natural numbers whose ratio was $\sqrt{2}$, but failed.
- The discovery of “irrational” ratios is attributed to Hippasus of Metapontum (5th cent. B.C.)
- After the $\sqrt{2}$ was revealed to be irrational, the Pythagoreans according to a legend killed Hippasus, not willing to believe this fundamental number could fail to be a ratio of integers.
- In Plato’s Republic it is stated that $\sqrt{50}$ is irrational.

$\sqrt{2}$

- Proof that $\sqrt{2}$ is irrational
  - $a/b = \sqrt{2}$ where both a and b are integers and the ratio a/b has been reduced to lowest terms. This means that a and b cannot both be even.
  - $a^2 = 2\,b^2$
  - a must be even since if it were odd then a^2 would be odd.
  - If a is even it can be written as 2c implying $4c^2 = 2\,b^2$ or $2c^2 = b^2$
  - By the same argument as above b must be even.
  - The assumption that $\sqrt{2}$ is rational leads to a contradiction.

Platonic Solids

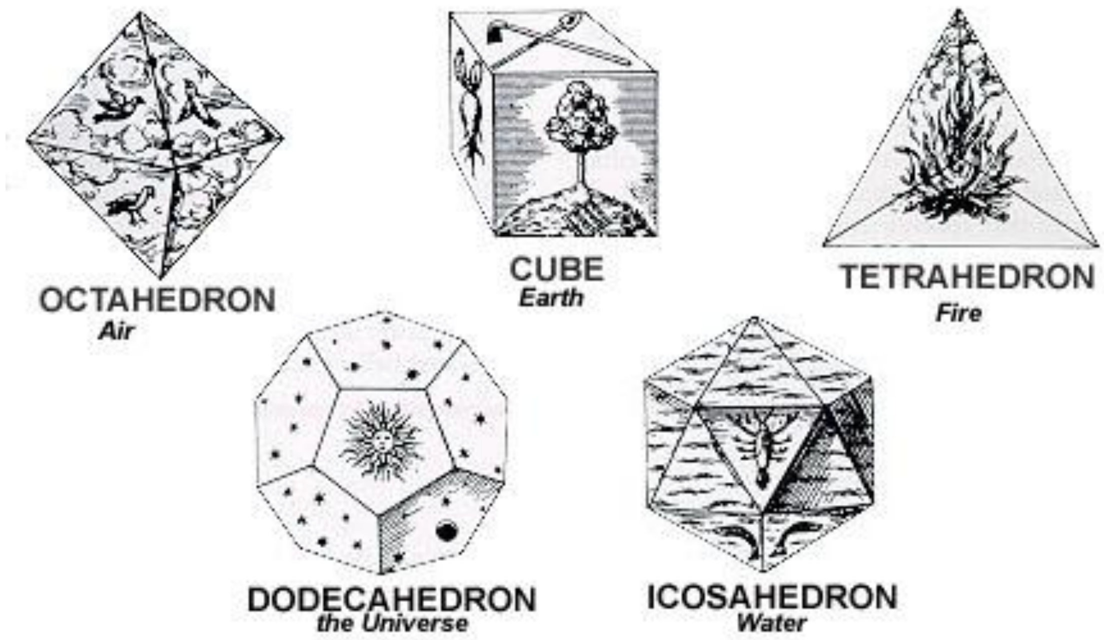
http://www.3quarks.com/en/PlatonicSolids/index.html

Pythagoras

- Tetrahedron – 4 equilateral triangles
- Cube (hexahedron) – 6 squares
- Dodecahedron – 12 equilateral pentagons

- Theaetetus (417 BC – 369 BC)
  - Octahedron – 8 equilateral triangles
  - Icosahedron – 20 equilateral triangles
- In book XIII of Euclid's elements it is shown that there are only five convex regular polyhedra
- So why are these called the Platonic solids?

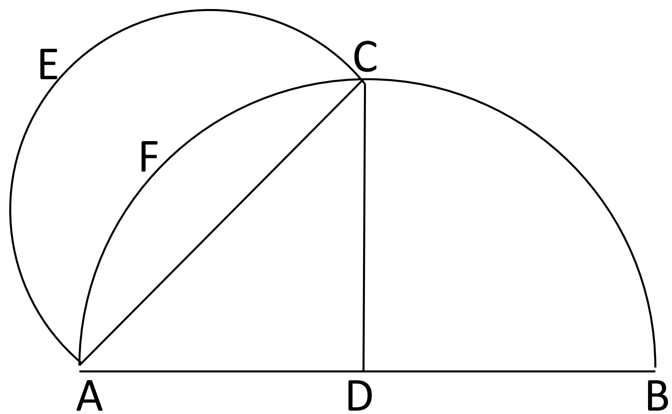
Platonic solids



Three Famous Geometry Problems of Antiquity

- These three problems were to be solved using a straight edge (not a ruler) and compass
  - Squaring the circle (Anaxagoras, 500 BC – 428 BC)
  - Doubling the cube (before Hippocrates of Chios, 470 BC – 410 BC)
  - Trisecting an angle (before Hippocrates of Chios)

Quadrature of Lunes



The area of the semicircle ACE equals the area of the quarter circle ADC.
Hence the area of the lune ACEF equals the area of triangle ADC

The fact that a shape that appears to be related to a circle can be shown to be equal in area to a triangle led folks to believe that the circle could be squared.

Famous Problems

- Squaring the circle
  - Squaring the circle involves constructing the square root of π which was shown to be impossible in 1882 when Ferdinand von Lindemann proved that π is transcendental.
- Doubling the cube (Delian problem) & trisecting an angle
  - Both these problems require finding the cube root of a quantity. In 1837 Pierre Wantzel showed the problems are unsolvable by compass and straightedge construction.
  - There are certain angles that can be trisected, e.g., 90 degrees but in general it cannot be done.

Beginning of the “Elements”

- Hippocrates of Chios
  - wrote the first Elements around 430 BC.
  - After Hippocrates at least four other Elements were written each improving on the previous versions
  - Euclid wrote his Elements in about 325 BC
- Hippocrates of Cos (460 BC – 375 BC)
  - father of rational medicine

Eudoxus of Cnidus (408 – 355 BC)

- Some consider him to be the greatest of the ancient mathematicians, second only to Archimedes
- Resolved the difficulty in comparing rational and irrational numbers
- Put the method of exhaustion on a rigorous basis
  - Similar to the limit concept of calculus
  - Proved the volume of a pyramid (cone) equals 1/3 vol. prism (cylinder) with same base and equal height
  - Area of circles go as the square of their diameters and the volume of spheres go as the cubes of their diameters

Euclid's Elements (300 BC)

The greatest mathematical textbook of all time

- Book I. The fundamentals of geometry: theories of triangles, parallels, and area.
- Book II. Geometric algebra.
- Book III. Theory of circles.
- Book IV. Constructions for inscribed and circumscribed figures.
- Book V. Theory of abstract proportions.
- Book VI. Similar figures and proportions in geometry.
- Book VII. Fundamentals of number theory.
- Book VIII. Continued proportions in number theory.
- Book IX. Number theory.
- Book X. Classification of incommensurables.
- Book XI. Solid geometry.
- Book XII. Measurement of figures.
- Book XIII. Regular solids.

Ptolemy I (323 – 285 BC)

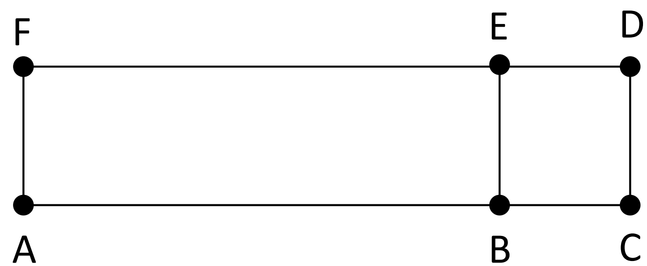
- Upon the death of Alexander the Great in 323 BC, the throne of Egypt fell to Ptolemy one of Alexander’s trusted commanders
- It is said that Ptolemy once asked Euclid if there was in geometry any shorter way than that of the Elements, and he replied that there was no royal road to geometry.

Euclid (Books 1 – 3)

- Book 1
  - Deals with basic definitions of points, lines and plane figures and related simple plane properties
- Book 2
  - Using the properties of rectangles and squares , introduces geometric algebra including solving the quadratic equation and the law of cosines. Shows how to convert any given polygon into a square of equal area.
- Book 3
  - Various properties of circles including Thales theorem and the properties of tangents

Euclid Book II (Geometric Algebra)

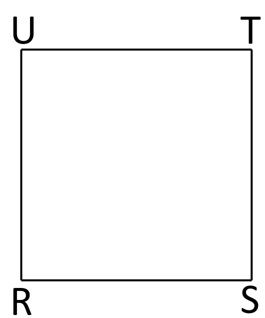
- Example: Given a line segment AB, find BC so that the area of the rectangle AD equals the area of square RT given that CD = BC .



What would we do?

AB = a, BC = x, CD = x, RS = s

$x(a + x) = s^2$, Solve for x.

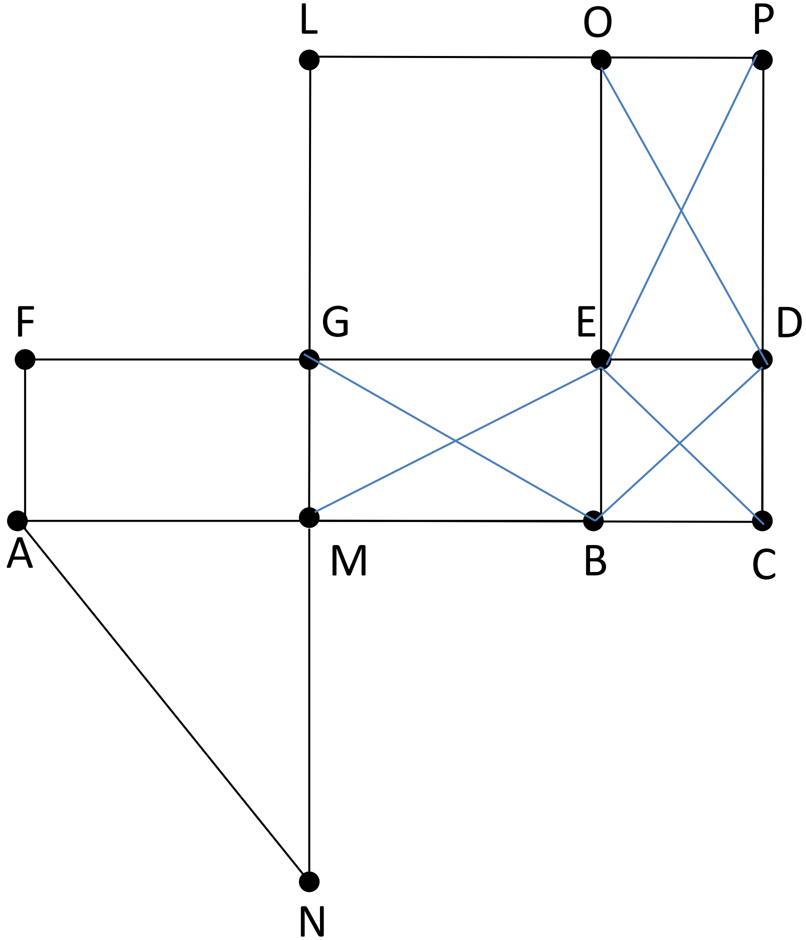


Example: a = 6, area of RT = 16.

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$; $x = 2$, $x = -8$

Example



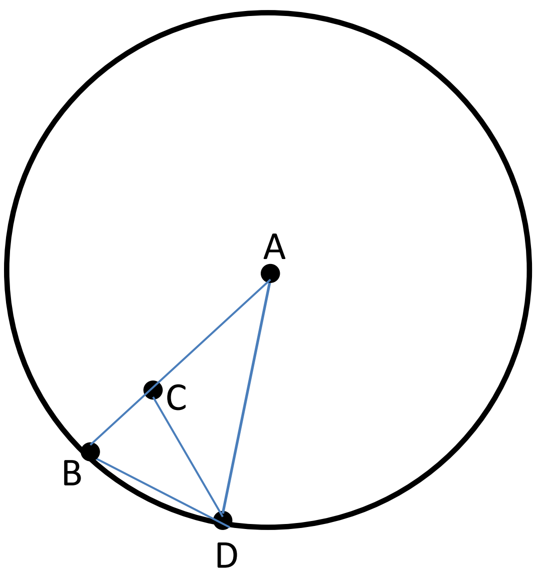
AB is original line segment.
M is the midpoint of AB.
MN is the side of the given square.
Construct the line segment MC equal to AN.
BC is the solution.

Check to see if correct
The sq on AM plus the sq on MN equals the sq on AN. The sq MP equals on AN. The sq on AM equals the sq LE. Sum of blue areas = sq on MN. AMGF = EDPO
Sq on MN = ACDF

Book 4

Inscribing and circumscribing triangles, squares, and regular polygons in and about circles

– Includes pentagon, hexagon, and 15-gon

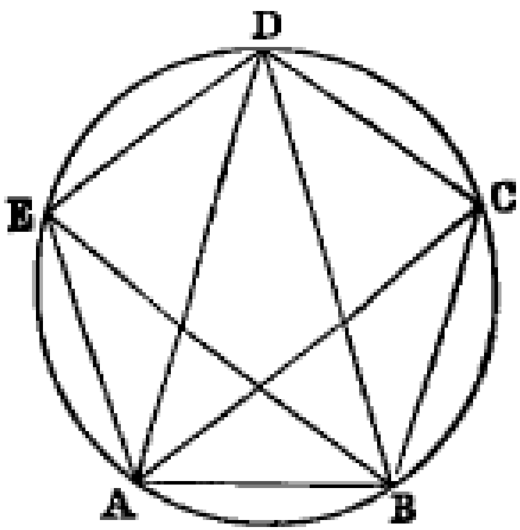


To Construct a regular pentagon, first construct an isosceles Δ with base angle 2x vertical angle

1) Draw the radius AB
2) Determine point C such that AB/AC = AC/BC
3) Construct BD = AC
4) From 2 & 3 AB/BD = BD/BC
5) Δ BDC is similar to Δ BAD
6) Δs BAD, BDC, and DCA are isosceles
7) Angle BDA = 2x angle BAD

Note: I have not followed the Euclid theorem sequence in the proof of the construction.

Inscribe a Regular Pentagon in a Circle



1) Construct the isosceles Δ of the previous slide in the circle (ΔABD)
2) Bisect angles ABD and DAB and extend to points E and C
3) Figure ABCDE is the inscribed pentagon
4) By inscribing an hexagon inside the circle starting at one of the pentagon's vertices a 15 – gon can be constructed.

Why is this interesting?

It took 2000 years until 1796 that Carl Friedrich Gauss proved that the regular 17-gon could be constructed.

The Theory of Numbers (Books 7 – 9)

- Book 7
  - Definitions: even, odd, prime, relatively prime (no common divisors), composite, perfect (6, 28, 496, 8128, ..) & others
  - Perfect numbers (number equals sum of divisors)
    - 6: divisors 1, 2, 3
    - 28: divisors 1, 2, 4, 7, 14
    - 496: divisors 1, 2, 4, 8, 16, 31, 62, 124, 248

  In Book 9 Euclid showed that $n = 2^{(m-1)} (2^m - 1)$ is perfect if $2^m - 1$ is a prime
  - Greatest Common Divisor (Euclid's Algorithm)
  - Least Common Multiple

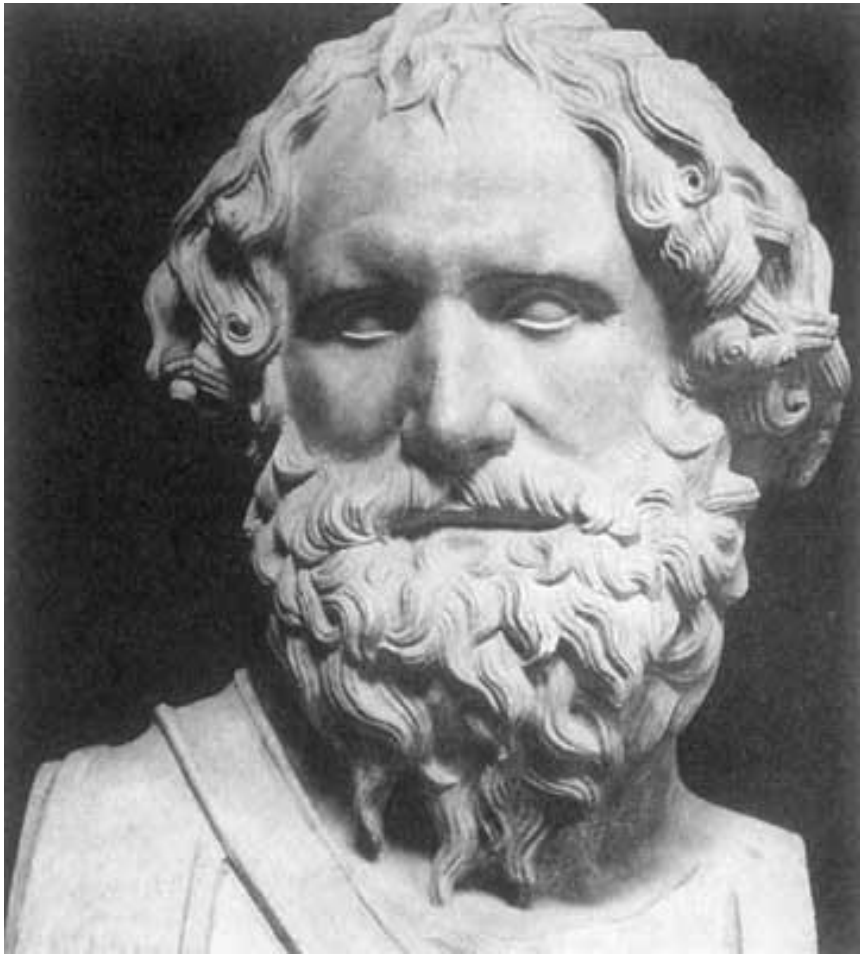
Book 8

- Book 8 deals primarily with geometric series
  - 2, 2x3, 2x3^2, 2x3^3, 2x3^4
    - 2, 6, 18, 54, 162
  - 32, 48, 72, 108, 162, 243
    - The fact that the first and last terms are relatively prime produces interesting consequences discussed by Euclid

Book 9 +

- Book 9 +
  - No prime can divide a product of other primes
    - A partial version of the fundamental theorem of arithmetic
  - There are an infinite number of primes
  - Sum of a geometric series
  - Euclid's rule for perfect numbers
  - Book X. Classification of incommensurables.
  - Book XI. Solid geometry.
  - Book XII. Measurement of figures.
  - Book XIII. Regular solids.
- Euclid's other books
  - Data, lost books (Pseudaria, Porisms, Conics, and Surface – Loci)

Archimedes

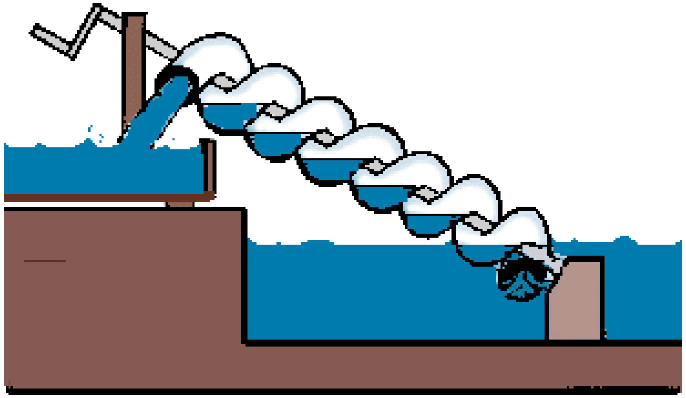


Archimedes

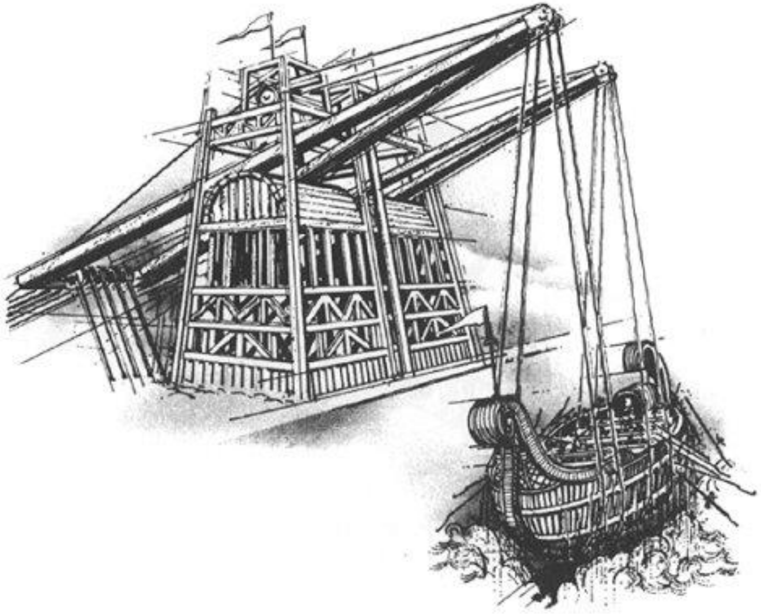
- Appears as a historical figure in 216 - 212 BC during the siege and capture of Syracuse
- Having got ready their blindages, missiles, and other siege material, they were in high hopes owing to their large numbers that in five days their works would be much more advanced than those of the enemy, but instead they did not reckon with the ability of Archimedes, or foresee that in some cases the genius of one man accomplishes much more than any number of hands. (Polybius)
- Killed when the city was sacked (age 75)

Archimedes and the Burning Mirror

Inventions



Screw Pump



The Archimedes Claw

Archimedes Mathematical Achievements

- Area of a circle (area of rt Δ with base = circumference and altitude = radius)
- Circumference = $2\pi r$, $3\frac{10}{71} < \pi < 3\frac{1}{7}$
- Area of a segment of a parabola
- Area of an ellipse
- Volume and surface area of a sphere
- Volumes of various "solids of revolution" obtained by rotating a curve about a fixed straight line.
- Law of the lever , center of gravity, hydrostatics

Archimedes Mathematical Achievements (cont.)

- One of the methods he used to find the areas, volumes and surface areas of many bodies was an early form of integration. This was considered his greatest mathematical invention, leading to the field of Calculus. To determine the area of sections bounded by geometric figures such as parabolas and ellipses, Archimedes broke the sections into an infinite number of rectangles and added the areas together.

Great Mathematicians

Gauss, Newton, Archimedes, Euler, Cauchy, Poincare, Riemann, Cantor, Cayley, Hamilton, Eisenstein, Pascal, Abel, Hilbert, Klein, Leibniz, Descartes, Galois, Mobius, Jacob, Johann and Daniel Bernoulli, Dirichlet, Fermat, Pythagoras, Laplace, Lagrange, Kronecker, Jacobi, Bolyai and Lobatchewsky, Noether, Germain, Euclid, Legendre

http://angelustenebrae.livejournal.com/15908.html?thread=37668

References

- I have borrowed liberally from:

1) Mathematics in the Time of the Pharaohs, Richard Gillings
2) The Historical Roots of Elementary Mathematics, Lucas Bunt, Phillip Jones, and Jack Bedient
3) An introduction to the History of Mathematics, Jamie Eves
4) Mathematical Thought from Ancient to Modern Times, Morris Kline
5) A History of Mathematics, Carl B. Boyer
6) The Mac Tutor History of Mathematics archive, University of St Andrews Scotland
7) A History of Greek Mathematics, Sir Thomas Heath
8) Wikipedia and other web sites
9) Famous Problems of Elementary Geometry, Felix Klein
10) Project Gutenberg's "First Six Books of the Elements of Euclid" by John Casey
11) Science & Music by Sir James Jeans (Dover)